- How many ordered triples of integers (a, b, c), with $a \geq 2$, $b \geq 1$, and $c \geq 0$, satisfy both $\log_a b = c^{2005}$ and $a + b + c = 2005$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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- **Solution (C)** The two equations are equivalent to $b = a^{(c^{2005})}$ and $c = 2005 - b - a$, so
$$c = 2005 - a^{(c^{2005})} - a.$$
If $c > 1$, then
$$b \geq 2^{(2^{2005})} > 2005 > 2005 - a - c = b,$$
which is a contradiction. For $c = 0$ and for $c = 1$, the only solutions are the ordered triples $(2004, 1, 0)$ and $(1002, 1002, 1)$, respectively. Thus the number of solutions is 2.

Difficulty:

NCTM Standard: Problem Solving Standard for Grades 9–12: represent and analyze mathematical situations and structures using algebraic symbols

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