

A triangle with side lengths in the ratio 3:4:5 is inscribed in a circle of radius 3. What is the area of the triangle?

- (A) 8.64 (B) 12 (C) 5π (D) 17.28 (E) 18

2007 AMC 10 A, Problem #14—

2007 AMC 12 A, Problem #10—

“The hypotenuse of the triangle is the diameter of the circle.”

Solution

Answer (A): Let the sides of the triangle have lengths $3x$, $4x$, and $5x$. The triangle is a right triangle, so its hypotenuse is a diameter of the circle. Thus $5x = 2 \cdot 3 = 6$, so $x = 6/5$. The area of the triangle is

$$\frac{1}{2} \cdot 3x \cdot 4x = \frac{1}{2} \cdot \frac{18}{5} \cdot \frac{24}{5} = \frac{216}{25} = 8.64.$$

OR

A right triangle with side lengths 3, 4, and 5 has area $(1/2)(3)(4) = 6$. Because the given right triangle is inscribed in a circle with diameter 6, the hypotenuse of this triangle has length 6. Thus the sides of the given triangle are $6/5$ as long as those of a 3–4–5 triangle, and its area is $(6/5)^2$ times that of a 3–4–5 triangle. The area of the given triangle is

$$\left(\frac{6}{5}\right)^2 (6) = \frac{216}{25} = 8.64.$$

Difficulty: Hard

NCTM Standard: Geometry Standard: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangle