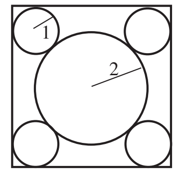
Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



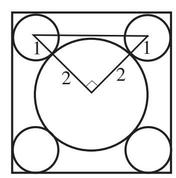
(A) 32 **(B)** $22+12\sqrt{2}$ **(C)** $16+16\sqrt{3}$ **(D)** 48
(E) $36+16\sqrt{2}$

2007 AMC 10 A, Problem #15— "Find the length of a side of the square by using the property of right angle in a circle."

Solution

Answer (B): Let s be the length of a side of the square. Consider an isosceles right triangle with vertices at the centers of the circle of radius 2 and two of the circles of radius 1. This triangle has legs of length 3, so its hypotenuse has length $3\sqrt{2}$.
The length of a side of the square is 2 more than the length of this hypotenuse, so $s = 2 + 3\sqrt{2}$. Hence the area of the square is

$$s^2 = (2+3\sqrt{2})^2 = 22 + 12\sqrt{2}.$$



OR

The distance from a vertex of the square to the center of the nearest small circle is $\sqrt{1^2+1^2} = \sqrt{2}$, and the distance between the centers of two small circles in opposite corners of the square is $1+4+1=6$. Therefore each diagonal of the square has length $6+2\sqrt{2}$, and each side has length

$$s = \frac{6+2\sqrt{2}}{\sqrt{2}} = 2+3\sqrt{2}.$$

The area of the square is consequently $s^2 = \left(2+3\sqrt{2}\right)^2 = 22+12\sqrt{2}$.

Difficulty: Hard
NCTM Standard: Geometry Standard: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
Mathworld.com Classification: Geometry > Plane Geometry > Circles