

Suppose that the number  $a$  satisfies the equation  $4 = a + a^{-1}$ . What is the value of  $a^4 + a^{-4}$ ?

- (A) 164    (B) 172    (C) 192    (D) 194    (E) 212

**2007 AMC 10 A, Problem #20—**

**“Square both sides of the first equation twice.”**

**Solution**

**Answer (D):** Squaring each side of the equation  $4 = a + a^{-1}$  gives

$$16 = a^2 + 2a \cdot a^{-1} + (a^{-1})^2 = a^2 + 2 + a^{-2}, \quad \text{so} \quad 14 = a^2 + a^{-2}.$$

Squaring again gives

$$196 = a^4 + 2a^2 \cdot a^{-2} + (a^{-2})^2 = a^4 + 2 + a^{-4}, \quad \text{so} \quad 194 = a^4 + a^{-4}.$$

**Difficulty:** Medium-hard

**NCTM Standard:** Algebra Standard: analyze change in various contexts.

**Mathworld.com Classification:** Calculus and Analysis > Special Functions > Powers > Squared